

10

RECORD

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FEDERAL SUPPLY SERVICE
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Book 10

Research & Computation Dairy

This book begun November 1968

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10
The following is a list of the

names of the persons who

W. K. C.

of the following names

of the following names

of the following names

of the following names

of the following names

of the following names
of the following names
of the following names

Space for Receipt of Cash

Bury Works disposed of

5/6/68 - 5/9/68

previous weeks { Willis ✓
Eccles ✓
Ezra ✓
Gordon Shepherd ✓

Wrote Annual Report

- ✓ letters to Per Anderson (Oslo meeting)
- ✓ Prof. Schenk (Buffalo Chair)
- ✓ Al Ochs (inquiry re retina)
- ✓ Rudomin (Mexico city, manuscript)
- ✓ J. Theod Bood - ~~accepting~~ to referee paper
- ✓ Billy Poffenberger
- ✓ Rosenblith (Bull. Biophys. Cong. dates)
- ✓ Steven Goldstein (U. Chicago Med. Student)
- ✓ Mc Intyre
- ✓ Azgo

✓ Talked with Maclean & collaborator

5/13/68 Talked with Peter Gouras on P potential

Danielli ✓ 6/7/68

at least.

Bury Work disposed of

5/6/68 - 5/9/68

previous weeks { Willis ✓
Eccles ✓
Ezra ✓
Gordon & Shepherd ✓

Wrote ✓ Annual Report

- letters to ✓ Per Anderson (Oslo meeting)
✓ Prof. Schenk (Buffalo Chair)
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✓ Azgo

✓ Talked with Maclean & collaborator

5/13/68 Talked with Peter Gouras on P potential

Write

MacGregor ✓ 5/27/68

Nastyuk (✓) 6/6/68

Willis (✓) 5/28/68

Burke (✓) 5/28/68

Spealman (✓) 6/13/68

Rudomin (✓) 6/6/68

? Phone F. O. Schmitt (✓)

Phone Reese & Gouros

Phone MacLean (✓)

Phone Phil re Rudomin
Symposium

Danielli (✓) 6/7/68

Checklist June 17-21

- Write MacGregor ✓
- Snell ✓
- Schmitt ✓
- MacKay ✓
- Leibovic ✓
- Science ✓
- Miss Leslie ✓
- Brookhart ✓
- Gordon Shepherd ✓

- Phone Machean ✓
- Write Nomination for John Evans ✓
- ~~Write Eccles~~
- Ask Dorothy for travel orders ✓

Look up Binghampton ✓

July 3, 1968 Checklist

Write Gordon ✓

Write Scheibels ✓

Write John Evans' nomination

Register for full. Congress

Congress Invitations

Order N.Y. Acad. reprints (find letter
quotation)

Order Cajal

Equalizing Time Constants

Sometime, note to Exp. Narel, with Mannon

also — Aitken & Shepherd

July 15, 1968

Today I spent the day adding halfway
down to finite cylinder series solutions.

Before Friday must

- Authorize check deposits
- Write Mackay
- ? Write Leibovic
- ? Work on Equalizing Time constants.
- John Evans nomination
- Terrier "
- Send manuscript to Terrier

7/16/68

Took care of

- ✓ Leiner Manuscript with covering letter
- ✓ Leibovic note
- ✓ NRP nomination for John Evans
- ✓ note to " "
- ✓ NPP nomination for Leiner

Still need to write Mac Kay

✓ & write Tsukahara

August 19, 1968

Professor D. M. Mackay
University of Keele

Dear Donald,

Please excuse my delayed reply. I have just returned from a much needed vacation; the first really relaxed one in several years. Today I am picking up the threads and returning to the busy life. To expedite it, I am typing this letter myself.

I shall be happy to participate in your work session and will plan to be ready with 10-15 minutes of introductory material on Monday morning. Your letter did not include your introductory program as an enclosure; consequently, I have not yet seen your outline.

Although I shall have to think a little more about exactly what to say, what I have in mind just now can be summarized as follows:

1. I am not an authority on evoked potentials in all their variety, but,
2. my experience encourages me to believe that it is possible to compute and thus predict the extracellular potentials generated by specified sequences of activity in specified populations of neurons;
3. such computations are simplest when the subpopulations of neurons are characterized by synchronous activity and by symmetric geometry, but
4. specified asynchrony and specified asymmetry could be provided for in more complicated computations.
5. The inverse problem, of inferring geometry and sequence of activity from the resultant extracellular potentials is much more difficult; in general, it has no unique solution.
6. Only by means of much auxiliary information on the underlying geometry and possible sequences of activity, would it seem feasible to seek valid solutions to this inverse problem.

Did you have in mind that I should give explicit details of the olfactory bulb study to ~~substantiate~~ illustrate my approach, or would you prefer to have those details deferred to a later point in the program?

Sincerely

P.S. I plan to remain in Brookline until Tuesday noon,
at least.

1-1

1-1



11/8/68 - Recap

Regular notetaking was suspended for almost a year. Things have been rather hectic

Note: Ravello in May & June 1967

Keele in September 1967 p.172 book 9

Udo's retinal detachment " Caianiello manuscript also explorations with John Evans

NRP Coding in Jan 1968

Atlantic City Symposium April 1968

* Completion of Ol Bulb Manuscript Feb-Mar 1968

Invitation from Oslo

Job offer for Buffalo 5/9/68

Jouras & P potential 5/14/68

Refereeing of MacGregor Rand Report

Rudomin informal referee & others

Bob Burke inquired about equalizing & 5/28/68

Beach in July

Intl. Physical Congress in August

Working on equalizing & manuscript in Sept.

Julian Jack visit in early October

Gave him 1st 15 pages of xerox draft

He has not used equalizing &

He used image method (reflections) for matching B.C.

also Johannesma in Sept. 25

Sept 16 - J. Neurophysiol galley proofs

Also in Sept - Talked with Phil, Bob & Dieter Lux

They decided not to go into complications that

I detailed in memo & computed

↑ 9/4/68

AC admittance & phase angle

11/8/68 - Recap

Regular meetings was suspended for almost a year. Things have been rather hectic

Note: Ravello in May & June 1967

Koole in September 1967

p. 172 books

Walt's natural detector "Camille manuscript" also experiments with different ones

1968
Feb 7 Mar

Completion of O. Ball's Manuscript
Ottavio in Symposium April 1968

Digitization from Oslo

for other projects 5/7/68

James & P. protocol 5/14/68

References of the paper and report

Richard in informal referees & others

Bob Burke inquired about operating 5/28/68

Break in July

Jack. Thomas' progress in August

Working on special 5 manuscript in Sept

Richard's book best in early October

Gene in 1st 15 pages of manuscript

He has not used equations &

He used image method (reflections) for matrix

also for same reason in Sept. 25

Sept 16 - J. Thompson's paper proofs

also in Sept - Tolstunov's but not the best

Richard's not to point comparisons that

& detected in memo & computer

10/1/68

AC submitted progress

11/8/68 recap



Also 9/6/68 - 9/10/68 wrote & tested WR 681T using remote station for real & imag parts for inf. cyl, finite cyl, & various p.

Then, for two weeks of October, pressed on with equalizing & manuscript.

FLW Missed NRP Ebooks (PO) (Wackay) Oct 20-22
 " Buffalo - had already declined
 Went to Mohawk Meeting Oct 27-30
 Refereeing Oct 17-25
 Letter from Tessard re IBRO Central Council
 More refereeing - resolved no more for a while
 Trying to work out arrangements for
 IBRO - Paris Dec. 16
 Scheibels - LA Feb 20
 Cowan here March 18-26
 Boulder - July 21 - Aug 8
 Rockefeller - Jan 9
 Oslo - Sept 15-17
must keep lid on

1st week of November 1968

Planning to resume & polish off equalizing &

(a) Still need to complete part with (b) and several dendrites allowing for different length.

(c) Then, probably separate eigenvalue part from complete solution part

11/8/88 recap

*

Also 11/10/88 - 11/11/88 wrote about WR 6811
want cement station for road & mag. part
for inf. copy, print off, & forward.
Then for two weeks of October, November on with
organizing & managing.

FWD

Missed NRP (Mackay) 08-20-22
N Buffalo - had already declined
Went to Mackay's Meeting 08-27-30
Referring
letter from President BRD Central Council
08-17-22

More referring - resolved no more for a while
trying to work out arrangements for
BRD - Paris Dec 16
Schedules - 1st Feb 20
Cannon was March 18 22
Boulder - July 21 - Aug 8
Rochester - Jan 9
Dale - Sep - 12-17
Must keep it on

1st week
of
November
1988

Plans to resume & post off organizing
(a) still need to complete part with P
(b) and several bundles
about for different length
(c) Then probably separate
organizing part
from complete section part

11/8/68

Today, before resuming work on equalizing ϵ manuscript, thought about the problem that I had been considering father Goldstein or Levine work on. Solved Crux here.

From N.Y. Acad paper when $K \neq 0$

PDE $\frac{\partial^2 V}{\partial z^2} + K \frac{\partial V}{\partial z} = V + \frac{\partial V}{\partial T}$

St.St. $\frac{d^2 v}{dz^2} + K \frac{dv}{dz} - v = 0$
(s+a)(s-b) (s^2 + Ks - 1)v = 0

-ab = -1
a-b = K

$V(z) = A e^{-az} + B e^{+bz}$

$r_1 = -a$ where $a = \frac{K + \sqrt{K^2 + 4}}{2} = 1 + \text{something small if } K \text{ is small}$

$r_2 = +b$ $-b = \frac{K - \sqrt{K^2 + 4}}{2} = -1 + \text{something small if } K \text{ is small}$

(note: for $K > 0$, $a > b > 0$ because $-ab = -1$ and $a - b = K$)

Separation of variables for transient.

$V(z, T) = U(z) e^{-(1+\gamma^2)T}$

substit in PDE

$(\frac{d^2 U}{dz^2} + K \frac{dU}{dz}) e^{-(1+\gamma^2)T} = (1 - 1 - \gamma^2) e^{-(1+\gamma^2)T}$

$\therefore \frac{d^2 U}{dz^2} + K \frac{dU}{dz} + \gamma^2 U = 0$

$(s^2 + Ks + \gamma^2)u = 0$

Can show we only want roots for $\gamma = 0$ and $\gamma^2 > K^2/4$

11/2/08

Today, before assuming work on separating variables, I had been considering factors of the form $\frac{gV}{gT} + K \frac{gV}{gT} = V + \frac{gV}{gT}$. I had been considering factors of the form $\frac{gV}{gT} + K \frac{gV}{gT} = V + \frac{gV}{gT}$. I had been considering factors of the form $\frac{gV}{gT} + K \frac{gV}{gT} = V + \frac{gV}{gT}$.

From N.Y. Good paper when $K \neq 0$

PDE $\frac{gV}{gT} + K \frac{gV}{gT} = V + \frac{gV}{gT}$

2.T. $\frac{gV}{gT} + K \frac{gV}{gT} - V = 0$
 $(2+K)(2-V) = 0$

$V(z) = A e^{-az} + B z + C$

where $a = \frac{K + \sqrt{K^2 + 4}}{2}$ and $b = \frac{K - \sqrt{K^2 + 4}}{2}$.
Note: for $K > 0$, $a > b > 0$, $a - b = -\sqrt{a - b} = K$.

Separation of variables for transient.

$V(z, T) = U(z) e^{-(1+\gamma^2)T}$

$(\frac{gV}{gT} + K \frac{gV}{gT}) e^{-(1+\gamma^2)T} = (1 - 1 - \gamma^2) e^{-(1+\gamma^2)T}$

$\frac{gV}{gT} + K \frac{gV}{gT} + \gamma^2 V = 0$

$(2 + K + \gamma^2)V = 0$

Can show we only want roots for $\gamma = 0$ and $\gamma^2 > K/4$

Now Here $U(z) = Ae^{-\alpha z} + Be^{-\beta z}$

where α & β are complex conjugates defined by

$$\alpha = \frac{K \pm \sqrt{K^2 - 4\gamma^2}}{2}$$

Note $\alpha\beta = \gamma^2$, $\alpha + \beta = K$

Notice, if $\gamma = 0$, this is case of uniform decay for both ends sealed. roots are zero & K

Zero root fits uniform decay e^0

But e^{-Kz} can be excluded as unable to satisfy the two sealed end B.C.

Similarly $0 < \gamma < \sqrt{K^2/4}$ gives real roots which cannot satisfy two sealed ends.

Because $\frac{dU}{dz} = -\alpha A e^{-\alpha z} - \beta B e^{-\beta z}$

Set zero at $z=0$ gives $\alpha A = -\beta B$

and at $z=L$ need $e^{-\alpha L} = e^{-\beta L}$

which is not possible for real $\alpha \neq \beta$

But it is possible when α & β are complex

$$\alpha = \frac{K}{2} \pm i\sqrt{\gamma^2 - K^2/4} \quad \text{for } \gamma^2 > K^2/4$$

let $\theta_m = \sqrt{\gamma_m^2 - K^2/4}$

or $\gamma_m^2 = \theta_m^2 + \frac{K^2}{4}$

from bottom right

$$\sum m = 0$$

Note that $\theta = 0$ would solve B.C. at $z=L$
but not B.C. at $z=0$

however, can have $\sin \theta L \neq 0$

provided that

$$\frac{-KB}{2} - \theta A = 0$$

$$\text{implying } \frac{B}{A} = -\frac{2\theta}{K}$$

But B.C. at $z=0$ already gave $\frac{B}{A} = \frac{K}{2\theta}$

$$\text{This would require } \frac{K}{2\theta} = -\frac{2\theta}{K}$$

$$4\theta^2 = -K^2$$

$$\text{or } \theta = iK/2 \quad \text{or } \frac{B}{A} = -i$$

But, if put this in topline of p. 12, get

$$e^{-Kz/2} \left\{ C_1 e^{-Kz/2} + C_2 e^{+Kz/2} \right\}$$

$$= C_1 e^{-Kz} + C_2 e^0$$

where B.C. require $C_1 = 0$

and leave C_2 arbitrary

$$\text{with } \theta = iK/2$$

$$\theta^2 = -\frac{K^2}{4}$$

$$y^2 = 0$$

Now, for these complex roots, we write

$$U(z) = e^{-Kz/2} \left\{ C_1 e^{+i\theta z} + C_2 e^{-i\theta z} \right\}$$

$$= e^{-Kz/2} \left\{ A \cos \theta z + B \sin \theta z \right\}$$

$$\frac{dU}{dz} = e^{-Kz/2} \left[\left(-\frac{KA}{2} + \theta B \right) \cos \theta z + \left(-\frac{KB}{2} - \theta A \right) \sin \theta z \right]$$

$$\frac{d^2U}{dz^2} = e^{-Kz/2} \left[\left(\frac{K^2A}{4} - K\theta B - \theta^2 A \right) \cos \theta z + \left(\frac{K^2B}{4} + K\theta A - \theta^2 B \right) \sin \theta z \right]$$

Testing this solution in $\left(\frac{d^2}{dz^2} + K \frac{d}{dz} + \gamma^2 \right) U = 0$

Coeff of $\cos \theta z$ is $e^{-Kz/2} \left[\frac{K^2A}{4} - K\theta B - \theta^2 A - \frac{K^2A}{4} + K\theta B + \gamma^2 A \right]$
or zero \downarrow
 $\theta^2 A + \frac{K^2A}{4}$

Similarly for coeff of $\sin \theta z$ set

$$e^{-Kz/2} \left[\frac{K^2B}{4} + K\theta A - \theta^2 B - \frac{K^2B}{4} - K\theta A + \gamma^2 B \right]$$

QED

or zero

$$\downarrow$$

$$\theta^2 B + \frac{K^2B}{4}$$

QED

$$\frac{dU}{dz} = 0 \text{ at } z=0 \text{ gives } 0 = 1 \left[\left(-\frac{KA}{2} + \theta B \right) \cdot 1 + 0 \right]$$

$$\therefore \frac{B}{A} = \frac{K}{2\theta} = \frac{KL}{2m\pi}$$

at $z=L$

$$0 = e^{-KL/2} \left[(0) \cos \theta L + \left(-\frac{KB}{2} - \theta A \right) \sin \theta L \right]$$

$$\therefore \sin \theta L = 0$$

$$\theta L = m\pi$$

$$\theta_m = m\pi/L$$

See left

eg. $K=1, L=1, e^{KL} = 2.718$ for $\frac{n}{m_0}$

$e^{-KL/2} = 0.606$

$e^{-KL/4} = 0.779$

$n=1, \frac{KL}{2m\pi} = \frac{1}{6.28}, \tau_0/\tau_1 = 1 + 0.25 + 9.87 = 11.12$

$n=2, \frac{KL}{2m\pi} = \frac{1}{12.56}, \tau_0/\tau_1 = 1 + 0.25 + 39.6 = 40.85$

Note sine term makes max contribution for $Z=L/2$

$0 = U(x) + \dots$

$A \sin \dots + B \cos \dots + C \sin \dots + D \cos \dots$

\dots

$0 = 1 - \dots$

$\frac{K}{2\pi m} = \dots$

\dots

Therefore, for two sealed ends, we find that for each n there is a term in $V(z)$

$$A_n e^{-Kz/2} \left\{ \cos(n\pi z/L) + \frac{KL}{2n\pi} \sin(n\pi z/L) \right\}$$

also, note $\theta_n^2 = \frac{n^2\pi^2}{L^2} = \gamma_n^2 - \frac{K^2}{4}$

or $\gamma_n^2 = \frac{n^2\pi^2}{L^2} + \frac{K^2}{4}$

∴ The solution can now be written

for $\frac{\partial^2 V}{\partial z^2} + K \frac{\partial V}{\partial z} = V + \frac{\partial V}{\partial T}$


for both ends sealed & arbitrary initial condition

$$V(z, T) = A_0 e^{-T} + e^{-Kz/2} \sum_{n=1}^{\infty} A_n \left\{ \cos(n\pi z/L) + \frac{KL}{2n\pi} \sin(n\pi z/L) \right\} e^{-\left(1 + \frac{K^2}{4} + \frac{n^2\pi^2}{L^2}\right) T}$$

where the effect of $K \neq 0$ can be seen at three points \odot & of course also would affect an initial steady state with current or voltage clamp.

Further problems: (1) other B.C.

(2) various I.C.

(3) two or more regions, eg.  +/or $E \neq 0$.

Therefore, for two leads \dots for both M there is a term in $V(\xi)$

$$A_m e^{-K \xi} \left\{ \cos(\alpha \xi) + \left(\frac{K}{\alpha} \right) \sin(\alpha \xi) \right\}$$

$$m=1, \quad \frac{K}{\alpha} = \frac{1}{25} = 0.04, \quad \alpha = 25, \quad \alpha \xi = 25 \times 1 = 25 \Rightarrow \alpha \xi = 11.12$$

$$m=2, \quad \frac{K}{\alpha} = \frac{1}{50} = 0.02, \quad \alpha = 50, \quad \alpha \xi = 50 \times 1 = 50 \Rightarrow \alpha \xi = 40.85$$

Note: \dots The solution can now be written

$$\text{for } \frac{\partial V}{\partial \xi} + K \frac{\partial V}{\partial \xi} = V + \frac{\partial V}{\partial \xi}$$

for both ends labeled \dots

$$V(\xi, T) = A_0 e^{-T} + e^{-K \xi} A_m$$

$$\left\{ \cos(\alpha \xi) + \left(\frac{K}{\alpha} \right) \sin(\alpha \xi) \right\} e^{-T} - \left(1 + \frac{K}{\alpha} \right) e^{-T}$$

where the effect of $K \neq 0$ can be seen at three points \dots ϕ \dots \dots

- (1) other B.C.
- (2) various I.C.



(3) two or more regions, e.g. \dots

11/27/68 day before Thanksgiving

Have just spent several days thinking about & preparing a statement on the "Role of Mathematical Theory in the Neurosciences". This prompted by plans of NRC Committee on Brain Sciences to discuss the scope of neurosciences in the light of a draft report (by Kuffler et al) which makes no reference at all to mathematical or biophysical theory.

During this same time, of course, I wrote Oxford & was making arrangements for Paris trip. Also worked up three more pages of equalizing E manuscript, and did other odd chores. Yesterday, got request from Science to review Proceedings of 4th Intl. Meeting of Neurobiologists (Stockholm 66) entitled "Structure and Function of Inhibitory Neural Mechanisms". This is the one where Bob Burke presented his data & Eccles dropped his old position. Tremendous number of items - big chore to review. Feel entitled to decline because of NRC & IBRO, as well as personal involvement with Burke-Eccles issue. Gordon Shepherd invited by Eccles to write review.

Musing about Reviews, textbooks, monographs.

The first book I should aim at should be "Experimental Theory of Neurons" "Introduction to Theoretical Neurophysiology"

With emphasis on [physiol → physics → math] building up intuitive grasp of this interrelation with the simplest examples. (over)

Also note that a comprehensive & selective
treatise would best be prepared by

someone who has conscientiously taught

in courses
Such material to students for several years.

I have not. My effort has been devoted

mainly to individual research,

with occasional lecturing & teaching.

Personal approach is natural & ^{more or less} unavoidable

11/27/68

Thoughts about

Experimental Neuro Theory

18

Introduction to Theoretical Neurophysiology

Preface emphasizing personal approach
belief in building intuitive grasp of
physical \rightarrow physics \rightarrow math

with simplest examples first.
Then move on to more complicated cases
& to general solutions.

~~Part I - Passive membrane~~

~~A. lumped patch~~

~~(1) intuitive approach, steady state as a
potential divider $V = \frac{G_a}{G_a + G_b}$ Comes later~~

~~B. ~~cylinder~~ infinite cylinder in medium of zero~~

Chapter I - lumped patch of membrane

A - passive

B - synaptic 1st pot. divider idea, then full math

C - active

Chapter II - ^{Nerve} Cylinders ~~of finite length~~ steady states

A - infinite length

B - finite length

C - soma dendrites

Chapter III - Transients in cylinders of infinite length

Chapter IV - Transients in cylinders of finite length

Experimental Human Theory

Autobiography to Researcher Neurophysiology

Abstract of the paper by [unclear]

Belief in evolution in [unclear]

by [unclear]

Some of the [unclear] target

Two more or less complete

Supplement to general

~~Part I of the [unclear]~~

~~A - [unclear]~~

~~Part II of the [unclear]~~

~~Part III of the [unclear]~~

~~Part IV of the [unclear]~~

Part I - [unclear]

A - [unclear]

B - [unclear]

C - [unclear]

Part II - [unclear]

Part II - [unclear]

A - [unclear]

B - [unclear]

C - [unclear]

Part III - [unclear]

Part IV - [unclear]

1/15/69
 Part V - Transients in Neurons

Could defer
 to a
 separate
 volume

Part VI - Extracellular field of spherical cell

Part VII - Extracellular field of cylinders

Part VIII - Extracellular field of soma + dendrite

Part IX - Extracellular field of pop. of neurons

Chap X - Spatio-temporal synaptic patterns
 may be earlier

Chap XI - Interacting neurons

Avoid compulsion to include everything.

Emphasize understanding of the simpler cases.

State more complicated cases & refer to literature

Footnote:

~~# readers~~ These readers who would like to see additional details included in a second edition are invited to write the ^{author or the} publisher specifying points where need is felt.

Part V - Transients in Nervous

Part VI - Cerebellar field of spherical cell

Part VII - Cerebellar field of excitatory

Part VIII - Cerebellar field of somatic

Part IX - Cerebellar field of prop. neurons

Part X - Extra-temporal synaptic patterns

Part XI - Interacting neurons

Cells refer to a separate volume

Good compiler to include everything.

Emphasize understanding of the simpler cases.

State more complicated cases & refer to literature

Footnote:

~~These~~ These researchers who would like to see additional details included in a second edition are invited to write the publisher's specifying points where need in fact.

1/15/69

Thurs.

(Last week at Rockefeller University)

22

Also, last week, started Steven Goldstein on problem from NY Acad. reprint — See p. 8 of this notebook. He worked out most of the same results independently.

This week, confronted with problem of non-orthog. i.e. our eigenfunctions are A_0

$$\text{and } A_n e^{-Kz/2} \sum \left\{ \cos \frac{n\pi z}{L} + \frac{KL}{2n\pi} \sin \frac{n\pi z}{L} \right\}$$

But, these eigenfunctions are not orthogonal.

At first, I thought of Churchill's paper which Jim had used to solve my earlier problem, but that was a boundary condition difficulty; furthermore, that general class does not include the present problem.

So we consulted Jim again

He pointed out two important things

(1) The factor $e^{-Kz/2}$ can be factored out because it does not depend upon n

(2) Our PDE separated to ordinary DE is not self adjoint, but it can be converted to self adjoint form as reviewed on next page.

1/12/69

(last week of Rachele's University)

Old books started Steven Johnson on problem
from N. Wood. report - Sep 8 of this week.
He worked out most of the same results independently.

This week, confronted with problem of non-orthog.

is, our eigenfunctions are A_0

$$\text{and } A_n \sim \sum_{k=1}^{\infty} \frac{1}{k^2} + \frac{1}{2n^2} + \dots$$

But, these eigenfunctions are not orthogonal.

At first, thought of Churchill's paper which
Jim had used to solve my earlier problem.
But that was a boundary condition difficulty;
Furthermore, that general class does not
include the present problem.

So we consulted Jim again.

He pointed out two important things
(1) The factor e^{-kx} can be factored out
because it does not depend upon n .

(2) Our PDE separated to ordinary DE
is not self adjoint but it can
be converted to self adjoint form
as reviewed on next page.

24

Self-Adjoint operators
Sturm-Liouville Equations

cf. Wazgenant & Murphy pp 253-255

Churchill p. 47 & p. 49
Fourier Series & BVP

Ince p. 215

Agnew - pp 251-277 esp p. 257

~~The general differential equation~~

From Ince

$$\cancel{p_0(x) \frac{d^2 y}{dx^2} + p_1(x) \frac{dy}{dx} + p_2(x) y}$$

The general homogeneous linear differential operator of 2nd order

$$L(u) = p_0 \frac{d^2 u}{dx^2} + p_1 \frac{du}{dx} + p_2 u \quad \left(\text{where } p_n(x) \text{ are funcs of } x \right)$$

is not self-adjoint in general, but it is self-adjoint if

$$p_0' = p_1$$

$$\text{Then } L(u) = \frac{d}{dx} \left(p_0 \frac{du}{dx} \right) + p_2 u$$

which is the operator studied by Sturm & Sturm-Liouville

* We can use the factor $\frac{C}{p_0} e^{\int \frac{p_1}{p_0} dx}$ to form a self-adjoint operator

$$\text{i.e. } \frac{C}{p_0} e^{\int \frac{p_1}{p_0} dx} L(u) \equiv \frac{d}{dx} \left\{ C e^{\int \frac{p_1}{p_0} dx} \frac{du}{dx} \right\} + \frac{p_2}{p_0} e^{\int \frac{p_1}{p_0} dx} u$$

where C is a constant to be chosen

If U_n & U_m are distinct eigenfunctions \rightarrow

$$\int_a^b p(x) U_n U_m dx = 0 \text{ is the orthogonality relation}$$

Whereas $\int_a^b U_n U_n dx \neq 0$

one could regard $(\int p U_n)$ $(\int p U_m)$
as the orthogonal functions.

yet Churchill does
call this orthogonality
of U_n & U_m

Note, since $U_0 = A_0$, this means
also that

$$\int_0^L e^{Kx} U_n dx = 0$$

$$\int_0^L e^{+Kx/2} \left\{ \cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right\} dx = 0$$

More specifically, from Churchill p. 47

$$\text{Given } u'' + f_1(x) u' + \{f_2(x) + \lambda f_3(x)\} u = 0$$

$$\text{use factor } r(x) = e^{\int f_1(x) dx} \quad \text{or } w(x) \text{ weighting factor}$$

$$\text{to form } \frac{d}{dx} \left[r(x) \frac{du}{dx} \right] + \{q(x) + \lambda p(x)\} u = 0$$

which is a Sturm-Liouville equation

$$\text{where } q(x) = r(x) f_2(x)$$

$$p(x) = r(x) f_3(x)$$

$$\text{In present problem } u'' + K u' + \lambda u = 0 \quad \lambda = \delta^2$$

$$\therefore r(x) = \int K dx = e^{Kx}$$

$$\therefore \frac{d}{dx} \left[e^{Kx} \frac{du}{dx} \right] + \lambda e^{Kx} u = 0$$

$$\therefore p(x) = e^{Kx}$$

\therefore we have that

$$\leftarrow \int_0^L e^{Kx} u_n u_m dx = 0$$

$$A_n A_m \int_0^L \left[\cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right] \left[\cos\left(\frac{m\pi x}{L}\right) + \frac{KL}{2m\pi} \sin\left(\frac{m\pi x}{L}\right) \right] dx$$

$= 0$

Charge on condenser dist. uniformly

On physical grounds

As already found for cylinder

A_0 is average of $f(z)$ over interval, weighted for surface area

$$\text{i.e. } \frac{\int_0^L f(z) \frac{dA}{dz} dz}{\int_0^L \frac{dA}{dz} dz}$$

where dA is increment in surface area.

$$\frac{dA}{dx} = 2\pi r n$$

$$\frac{dA}{dz} = 2\pi r n \frac{dx}{dz}$$

$$\propto 2\pi r n^{3/2}$$

$$\propto e^{kz}$$

$$\frac{dx}{dz} \propto r^{1/2}$$

P. 1079 of NY Acad.

because we here take $\frac{ds}{dx} = 1$

would even apply for more general $\frac{ds}{dx}$

$$\therefore A_0 = \frac{\int_0^L f(z) e^{kz} dz}{\int_0^L e^{kz} dz}$$

1/15/69 continued Now

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n e^{-Kx/2} \left\{ \cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right\}$$

$$e^{Kx} f(x) = e^{Kx} A_0 + \sum_{n=1}^{\infty} A_n e^{+Kx/2} \left\{ \cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right\}$$

Integrate each term ~~over~~ over interval,

$$\text{get } \int_0^L e^{Kx} f(x) dx = A_0 \int_0^L e^{Kx} dx + \text{O for each term of series}$$

$$\therefore A_0 = \frac{\int_0^L f(x) e^{Kx} dx}{\int_0^L e^{Kx} dx}$$

↑
See bottom p. 25

as was already deduced on physical grounds
(see left.)

Here x corresponds to z of NY acid problem

$$\text{Also } A_n = \frac{\int_0^L f(x) e^{+Kx/2} \left\{ \cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right\} dx}{\int_0^L \left\{ \cos\left(\frac{n\pi x}{L}\right) + \frac{KL}{2n\pi} \sin\left(\frac{n\pi x}{L}\right) \right\}^2 dx}$$

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On physical grounds

derivation for $\frac{1}{r^2}$

intensity, weighted for surface area

$$\int_0^{\infty} \left(\frac{1}{r^2} \right) A_{\text{shell}} dr = \int_0^{\infty} \left(\frac{1}{r^2} \right) 4\pi r^2 dr = 4\pi \int_0^{\infty} dr = \infty$$

intensity is constant

$$\int_0^{\infty} e^{-kr} dx = \frac{1}{k} [-e^{-kr}]_0^{\infty} = \frac{1}{k} (0 - (-1)) = \frac{1}{k}$$

$$\int_0^{\infty} e^{-kr} r^2 dr = \frac{2}{k^3}$$

always kept for later reference

Next concept of Hydrogen

$$\int_0^{\infty} \left(\frac{1}{r^2} \right) A_{\text{shell}} dr = \int_0^{\infty} \left(\frac{1}{r^2} \right) 4\pi r^2 dr = 4\pi \int_0^{\infty} dr = \infty$$

1/16/69

Because $\int e^{ax} \cos px dx = \frac{e^{ax} (a \cos px + p \sin px)}{a^2 + p^2}$
 $\int e^{ax} \sin px dx = \frac{e^{ax} (a \sin px - p \cos px)}{a^2 + p^2}$

See p. 309 of 12th ed. of CRC Math Tables.

It follows that

$$\int_0^L e^{Kx/2} \left\{ \cos \frac{n\pi x}{L} + \frac{KL}{2n\pi} \sin \frac{n\pi x}{L} \right\} dx =$$

$$\Rightarrow \left[\frac{e^{Kx/2}}{\left(\frac{K}{2}\right)^2 + \left(\frac{n\pi}{L}\right)^2} \left(\frac{K}{2} \cos \frac{n\pi x}{L} + \frac{n\pi}{L} \sin \frac{n\pi x}{L} + \frac{KL}{2n\pi} \sin \frac{n\pi x}{L} - \frac{n\pi KL}{K^2 + \frac{4n^2\pi^2}{L^2}} \cos \frac{n\pi x}{L} \right) \right]_0^L$$

= 0

New denominator of A_n

$$\int_0^L \left\{ \cos \left(\frac{n\pi x}{L} \right) + \frac{KL}{2n\pi} \sin \left(\frac{n\pi x}{L} \right) \right\}^2 dx =$$

$$(\cos a + b \sin a)^2 = \cos^2 a + 2b \cos a \sin a + b^2 \sin^2 a$$

$$a = \frac{n\pi x}{L}$$

$$b = \frac{KL}{2n\pi}$$

$$\frac{1}{n\pi} \left[\frac{1}{2} \sin a \cos a + \frac{a}{2} + b \sin^2 a - \frac{b^2}{2} \cos a \sin a + \frac{ba}{2} \right]$$

$$= \frac{1}{n\pi} \left(0 - 0 + \frac{n\pi}{2} - 0 + 0 - 0 + 0 - 0 + \left(\frac{KL}{2n\pi} \right)^2 \frac{n\pi}{2} \right)$$

$$= \frac{L}{2} \left(1 + \left(\frac{KL}{2n\pi} \right)^2 \right) \quad \text{or} \quad \left(\frac{n\pi}{2} + \frac{(KL)^2}{8n\pi} \right) \frac{L}{2}$$

$$\frac{C_1 \omega^2 + C_2 \omega + C_3}{\omega^2 + 4\omega + 7} = \frac{C_1(\omega + 2) + C_2(\omega + 1) + C_3}{\omega^2 + 4\omega + 7}$$

equating coefficients

$$0 = \frac{4}{15} \Rightarrow 0.266$$

$$0 = 1 + 1 = 2$$

$$0.266 = \frac{0.503}{4} = 0.12575$$

$$\frac{C_1 \omega^2 + C_2 \omega + C_3}{\omega^2 + 4\omega + 7} = \frac{C_1(\omega + 2) + C_2(\omega + 1) + C_3}{\omega^2 + 4\omega + 7}$$

$$C_1 \omega^2 + C_2 \omega + C_3 = C_1 \omega^2 + (2C_1 + C_2)\omega + (2C_1 + C_2 + C_3)$$

$$C_1 = C_1$$

$$C_2 = 2C_1 + C_2$$

$$C_3 = 2C_1 + C_2 + C_3$$

for $C_n = 2C_0 (-1)^n$ for δ input at $R=2$

$$G(t_p) = \frac{2[1+4(\frac{T}{2})^2]}{[+2(1+\frac{T}{2})^2 - G]^3} - \frac{2[1+9(\frac{T}{2})^2]}{[+2(1+\frac{T}{2})^2 - G]^8} + \dots$$

$$\approx \frac{(\pi/2)^2}{[1+(\pi/2)^2]^3}$$

e.g. if $\pi/2 = 2$ get $\approx \frac{4}{125} = 0.03$

3 parts per
thousand
in arg.
of \ln

compared with $2[1+4] = 10$

9.97

i.e. for this case $t_p/\tau_{em} = (\frac{1}{2})^2 \ln\{10\}$

$$= \frac{2.303}{4} = 0.576$$

$$\rightarrow \frac{2.2996}{4} = 0.575$$

note that $e^{-(n^2-1)(\frac{T}{2})^2 t_p/\tau_{em}} \approx \left\{ \frac{C_1}{C_0} [1+(\frac{T}{2})^2] \right\}^{-C_1 t_p/\tau_{em}}$

$-G(t_p)$

$$\therefore G(t_p) = \sum_{n=2}^{\infty} \frac{C_n}{C_0} \frac{[1+(\frac{nT}{2})^2]}{\left\{ \frac{C_1}{C_0} [1+(\frac{T}{2})^2] - G \right\}^{(n^2-1)}}$$

$$\approx \frac{C_2}{C_0} \frac{[1+4(\frac{T}{2})^2]}{\left\{ \frac{C_1}{C_0} [1+(\frac{T}{2})^2] \right\}^3} + \text{smaller terms}$$

Peak of decay transient

34

Recap & Generalize earlier results from Dec 1967

$$V(t) = C_0 e^{-k_0 t} + C_1 e^{-k_1 t} + \sum_{n=2}^{\infty} C_n e^{-k_n t}$$

$$\dot{V}(t) = -k_0 C_0 e^{-k_0 t} - k_1 C_1 e^{-k_1 t} - \sum_{n=2}^{\infty} k_n C_n e^{-k_n t}$$

When $\dot{V}(t) = 0$, $t = t_p$, and we can write

$$e^{-(k_0 - k_1)t_p} = -\frac{k_1 C_1}{k_0 C_0} - \sum_{n=2}^{\infty} \frac{k_n C_n}{k_0 C_0} e^{-(k_n - k_1)t_p}$$

or \ln both sides

$$(k_1 - k_0)t_p = \ln \left\{ -\frac{k_1 C_1}{k_0 C_0} - \sum_{n=2}^{\infty} \frac{k_n C_n}{k_0 C_0} e^{-(k_n - k_1)t_p} \right\}$$

Now for cylinder of electrotonic length, L , and sealed ends,

$$k_n = \left[1 + \left(\frac{n\pi}{L} \right)^2 \right] k_0 = \frac{1 + \left(\frac{n\pi}{L} \right)^2}{\tau_{em}}$$

$$k_1 - k_0 = \left(\frac{\pi}{L} \right)^2 k_0 = \frac{(\pi/L)^2}{\tau_{em}}$$

$$k_n - k_1 = (n^2 - 1) \left(\frac{\pi}{L} \right)^2 k_0$$

$$\frac{k_1}{k_0} = 1 + \left(\frac{\pi}{L} \right)^2$$

$$\frac{k_n}{k_0} = 1 + \left(\frac{n\pi}{L} \right)^2$$

$$\therefore \frac{t_p}{\tau_{em}} = \left(\frac{L}{\pi} \right)^2 \ln \left\{ -\frac{C_1}{C_0} \left[1 + \left(\frac{\pi}{L} \right)^2 \right] - G(t_p) \right\}$$

where $G(t_p) = \sum_{n=2}^{\infty} \frac{C_n}{C_0} \left[1 + \left(\frac{n\pi}{L} \right)^2 \right] e^{-(n^2 - 1) \left(\frac{\pi}{L} \right)^2 t_p / \tau_{em}}$
 (usually small)

see left

Book of copy transcript

27

Prop. of ...

$$V(x) = -k_0 \cos x - k_1 \cos 2x - k_2 \cos 3x - \dots$$

$$V(x) = -k_0 \cos x - k_1 \cos 2x - k_2 \cos 3x - \dots$$

Example: ...
 $\frac{1}{2} = \frac{1}{2} \Rightarrow \dots$
 $\frac{1}{2} = \frac{1}{2} \Rightarrow \dots$

$$(k_1 - k_0) \frac{1}{k_0} = \dots$$

Now far cylinder of electrostatic length, ...

$$k_m = \frac{1}{1 + \left(\frac{m\pi}{L}\right)^2} \left[\frac{1}{k_0} + \dots \right]$$

$$\frac{1}{k_0} = 1 + \left(\frac{m\pi}{L}\right)^2 \left[\frac{1}{k_0} - \dots \right]$$

where $G(x) = \dots$
 (amplitude)

Now, for initial condition weighted most in outer half of cylinder, we get, for $V(t) = z = 0$, that C_1 has opposite sign to C_0 & formula can be used as stated

$$\text{Note } \frac{C_1}{C_0} = \frac{2 \int_0^L F(x) \cos\left(\frac{\pi x}{L}\right) dx}{\int_0^L F(x) dx}$$

for $F(x) = \delta(L-x)$, get $\frac{C_1}{C_0} = \frac{-2}{1} = -2$

for $F(x) = \begin{array}{c} \text{---} \\ \text{A} \quad \text{L} \end{array}$, get $\frac{C_1}{C_0} = \frac{-2 \sin(\pi A/L)}{(\pi/L)(L-A)}$

for $F(x) = \delta(A-x)$, get $\frac{C_1}{C_0} = 2 \cos\left(\frac{\pi A}{L}\right)$

For above $\frac{t_p}{t_m} \approx \left(\frac{L}{\pi}\right)^2 \ln \left\{ \frac{2[1+(\frac{L}{L})^2] \int_0^L F(x) \cos\left(\frac{\pi x}{L}\right) dx}{\int_0^L F(x) dx} \right\}$

for $\delta \longrightarrow \left(\frac{L}{\pi}\right) \ln \left\{ +2[1+(\frac{L}{L})^2] \cos(\pi A/L) \right\}$

for $\begin{array}{c} \text{---} \\ \text{A} \quad \text{L} \end{array} \cdot \text{ get } \left(\frac{L}{\pi}\right) \ln \left\{ \frac{-2[1+(\frac{L}{L})^2] \sin(\pi A/L)}{(L-A)(\pi/L)} \right\}$

Simplest is δ at L , then get $\left(\frac{L}{\pi}\right) \ln \left\{ -2[1+(\frac{L}{L})^2] \right\}$

Now for initial condition we get $V(0) = 2 = 0$, that is, C_1 has opposite sign to C_2 formula can be used as stated

Note $\frac{C_1}{C_2} = \frac{\int_0^L F(x) \cos(\frac{\pi x}{L}) dx}{\int_0^L F(x) dx}$

Example

for $F(x) = \delta(L-x)$, get $\frac{C_1}{C_2} = \frac{1}{1} = 1$

for $F(x) = \sqrt{x}$, get $\frac{C_1}{C_2} = \frac{\int_0^L \sqrt{x} \cos(\frac{\pi x}{L}) dx}{\int_0^L \sqrt{x} dx}$

for $F(x) = \delta(A-x)$, get $\frac{C_1}{C_2} = \frac{1}{\cos(\frac{\pi A}{L})}$



$\int_0^L F(x) \cos(\frac{\pi x}{L}) dx$ for $F(x) = \delta(x)$

$\int_0^L F(x) \cos(\frac{\pi x}{L}) dx$ for $F(x) = \delta(x)$

$\int_0^L F(x) \cos(\frac{\pi x}{L}) dx$ for $F(x) = \delta(x)$

$\int_0^L F(x) \cos(\frac{\pi x}{L}) dx$ for $F(x) = \delta(x)$

2/6/69

Just spent last several weeks

- (a) getting Steven Goldstein started
- (b) writing final draft of equalizing τ paper
- (c) included working out θ dependence appendix and also voltage clamp

\rightarrow multiple cylinders to one soma.

Most of it has been proofread & xerayed today.

Still need

- Conclusions
- Abstract
- Fig. 1 & Fig legend

Papers

I - Equalizing τ

II - Explicit Solutions

III - Solutions for peaks etc.

IV - AC admittance

V - cylinder in applied external field

Also - Extracellular Potentials

with John Evans & EEG - Congress
& EEG Handbooks

Also - preprint for Scheibel work session

Recently refereed & also wrote help to Ill. Grad Student (Whitling)

2/10/8

Just about that several weeks

- (a) getting steam galvanic started
 - (b) writing final draft of apparatus & paper
 - (c) included working but dependent appendix and also voltage clamp
- Must of it has been proof read & revised today

Still need [Cerebellum
Electro]
for 1st & 2nd paper

- Paper
- I - Cerebellum
 - II - Cerebellum
 - III - Cerebellum for paper etc.
 - IV - AC observations
 - V - Cerebellum in applied external field

also - Physiological observations
with John Brown & EEG - Cerebellum
& EEG - Cerebellum

also - report for scheduled work session

Recently referred & also write up to the first session (with)

May 19, 1969

40

Last two months have been hectic
End of February included funeral (Howard Freed)
& Scheibel Symposium
& visit to John Evans

first half of March devoted to chores
Rest of March, all of April & 1st half of May
devoted mainly to working out details &
writing up new paper:

Paper II { Distributions of Potential in Cylindrical
Coordinates and Time Constants for a
Membrane Cylinder. which grew

out of what was at first an appendix of
equalizing time constant paper.

last Friday - May 16 - completed original &
5 xerox copies of what I hope is final
draft.

Copies being read by John Krugel
Jose
Zinn

Paper I Now need to do final cleanup of Equalizing
Time constant paper which was
essentially finished in February
before going to Scheibel Symposium.
When ready: Send copies to Jack & Redman.

May 19, 1961

Last week politely declined

① Teuber - IBRO Symposium Committee

② Yphantis - Biophysical Soc. Council
Nomination

District Directors of Potential in Cylindrical
Coordinates and Time Constants for a
Membrane Capacitor. About 1960

out of what was at first an appendix of
qualifying time constant papers.

let Friday - May 16 - completed original
draft of report of what there is final

capacitors read by
John
John

Now need to do final cleanup of qualifying
time constant papers with
eventually filed in February
before going to Robert
and paper to Robert

May 26, 1969

Now have 8 xeroxes of corrected Paper I
5 " " " " " " Paper II
I will make a few final corrections to original of II
before running off more xeroxes

Clearance copies going to NIAID Clearance today.
Should get Figs from Ad Dept tomorrow.
↑ two for paper II.

Paper I
original

5/23/69

- copy #1 Mexes mailed to Redman Boulder
- #2 desk of Jack 7/11/69
- #3 John Evans
- #4 Ringzell copy to Boulder (Master)
- #5 desk
- #6 Haguis
- #7 Home
- #8 - Clearance

include
5/26/69
corrections

Paper II
original

Redman
mailed
Boulder

5/16/69

- copy #1 Ringzell
- #2 Zinn → Clearance
- #3 7/11/69 Jack
- #4 Mexes John Evans
- #5 Master desk Correction
- #6 K.S. Cole 9/25/69
- #7 desk
- #8 Haguis
- #9 Home

5/26/69

? Copies for Julian Jack
Stephen Redman
Steven Goldstein

means updated
6/3/69
Richard
Footnotes 1 & 7
p. 41 Lorente

Made 4 more xeroxes 9/3/69 of corrected
gave one to Bob Burke
Send one to Oslo, brought back: Cole 9/25/69

submitted 6/4/69 submitted 6/4/69
To Biophysical J.

May 20, 1969

I will make for first question to original II
 before running off more xeroxes
 Clearance copies sent to NIAID clearance today.
 Search of file from 62-577 forward.
 → back to paper II.

Paper II		Paper I	
Original		Original	
copy #1	copy #1		
#2	#2		
#3	#3		
#4	#4		
#5	#5		
#6	#6		
#7	#7		
#8	#8		
Home	Home		

I have copies of all of records
 from one to the other
 Search on a file brought back: 62-577
 To submit 6/1/69
 Submit 6/1/69

Have been very busy

Boulder in July & August (1969)

Norway in September (1969)

Worked on Oslo manuscript after return

Handicapped by no Secretary
got overtime typing.

Brothup Jr. Galley proofs came Oct 9, 1969
returned Oct 11, 1969

Finally completed Oslo manuscript typing
& xeroxing
Oct 15, 1969

- 8 xerox copies
- 1 sent to Oslo with original
- 1 home
- 1 master
- 1 Hagins for referee
- 1 Burke for referee
- 3 in cabinet

5/9/69

5/13/69

6/5/69

- ① Dept to demilitarize
- ② Explos Action Prot. Model

- ③ Use of ...
- ④ ...

Have been very busy

Boulder in July & August (1969)

Norman in September (1969)

Worked on Bob's manuscript after return
identified in no secretary
got overtime typing.

Richard J. Galley proofs come Oct 7, 1969
returned Oct 11, 1969

Oct 12, 1969
Finally completed Bob's manuscript typing
& revising



- 8 xerox copies
- 1 sent to Bob with original
- 1 home
- 1 master
- 1 Hooper for reference
- 1 bank for reference
- 3 in cabinet

February 24, 1970

Starting to write up dendritic spine work with John Krügel.

Take time for a moment to go over loose notes on development.

We started talking in March or April 1969

4/4/69 John had memo on his version of compartmental calcs. for straight chain

4/31/69 John produced a few twig results. ^{& memo of equations} I urged twig. calc.

5/8/69 my memo proposing specific computations.

5 cpts. $\Delta z = 0.2$ plus one twig at end.

* use different values for coupling R calc. steady state

& for E, i & V pulses.

made qualitative predictions.

* { Twig V increase with R_c increase
Soma V decrease with R_c increase

5/9/69 some more detailed suggestions

John produced some computed results
resistance ratios

5/13/69 John's followup memo on resistance ratios

6/5/69 my memo on possible collaborative projects that could lead to publishable papers.

① Inputs to dendritic spines

② Explore Action Pot. Model

③ Use Cpt. & Act. Pot. model to compute EC Pot.

④ Small set of connected neurons

February 24, 1970

Starting to write up dendritic spine work
with John King.

Take time for a manuscript to go over
notes on development.

We started talking in March or April 1969
John Westerman on his version of compartmental
calc. for straight chain

4/31/69 John produced a few things
2/8/69 my name proposing specific computations.
I got $\Delta T = 0.2$ plus one thing at end.

* use different values for coupling R
calc. steady state
for E, I & V pulses.
made qualitative predictions.

* I gave V curves with R. minor
* I gave V curves with R. minor

2/7/69 some were detailed suggestions
John produced some computed results
resistance ratios

2/12/69 John's followup memo on resistance ratios

6/2/69 my memo on possible alternative proposals that
could lead to publishable papers.

- ① Input to dendritic program
- ② Input to dendritic program
- ③ Use of 4- or 5- compartmental model
- ④ Small set of computed results

John Ruizell was away in June 1969
 we worked some in July
 but then I had to prepare for Boulder
 & Oslo.

7/3/69 Ruizell memo summarizing status of spine
 problem
 Asks a number of values & questions.
 Spine dimensions, etc.
 Some outline for a paper: still opt. al

7/8/69 My memo for preparation of a short paper.
 spine dimensions
 specific answers to (7/3/69)

I propose (A) gemmule coupled to
 (B) neuron "green's Fern"
 Suggest simultaneous calc.

7/9/69 New plan ↑ with diagrams
 where I state the kinds of solutions needed

This led to John's work collected in hard binder

(7/30/69) Convolution calc.
 Solutions of BVP

8/8/69 Convergence tests of series

8/19/69 Summary of Spine Calcs.

which is a shot at a write up

Interrupted by Oslo & Manuscripts for Oslo &
 Boulder

November/11/69 New memo on spines

John Ringell was away in June 1969
we worked some in July

but then I had to prepare for Boulder
& Oslo.

7/3/69 Ringell means summarizing status of spine
problem

later a number of values & questions
Spine dimensions, etc.
some outline for a paper: still opt. al

7/8/69 My memo for preparation of a short paper.
spine dimensions
specific answers to (7/3/69)

Propose (A) examples coupled to
(B) version "years for"
Support simultaneous calc.

7/9/69 New plan I into diagrams
want state the kind of solution needed

This led to John's work collected in hand binder
(17/20/69) Caroubution calc.

20/2/69 20/2/69
8/8/69 Comparison table of spine

8/17/69 Summary of spine Calc.
which is a short etc. with sp

interrupted by Oslo & Manuscript for Oslo &
Boulder

November 11/69 New memo on spine

Nov. 11, 1969

Notes on Dendritic Spines

Re: Strategy to wrap up computations & prepare for publication

Major variables

 R_h = spine head membrane resistance R_{st} = spine stem " " $R_{ds} = R_{ts} = R_{bin}$ input resistance R_N = whole neuron.

steady state & transient.

Discuss:

(a) degree of isolation

(b) enhanced non-linearity within spine

(c) " " linearity between " "

(d) effect of changing R_{st} Ball park values $R_h \sim 10^9$ to $10^{10} \Omega$ $R_{st} \sim 10^7$ to 10^8 $R_{bin} \sim \begin{cases} 10^6 \text{ to } 10^8 & \text{for single cyl.} \\ 10^7 \text{ to } 10^9 & \text{for branch} \end{cases}$

(p.6) predictions

Thoughts about graphs

Wrote soln

11/18/69 Solved N cylinders coupled at origin

11/20/69 Wrote solution for one order of branching and generalized to many orders.

12/1/69 what happens when $Ed^{3/2}$ not const.?

Nov. 11, 1969

Notes on dendritic spines
his strategy to wrap up computations
& prepare for publications

Major variables

- R_m = spine head membrane resistance
- R_s = spine stem
- R_p = spine pedicle resistance
- R_i = axial membrane

steady state & transient

Issues:

- (a) degree of isolation
- (b) enhanced non-linearities within spine
- (c) " " " " linearity between
- (d) effect of changing R_s

Ball park values

- $R_m \sim 10^7$ to $10^{10} \Omega$
- $R_s \sim 10^5$ to 10^8
- $R_p \sim \{ 10^6 \text{ to } 10^9 \text{ } \Omega \}$ (unq. of)
- $R_i \sim 10^7$ to $10^9 \Omega$ (per branch)
location

(p.d.) publications

Thought about graphs

White 1969

11/18/69 solved N equations coupled at origin
11/20/69 White solution for order of branching
and generalized to many orders.
12/1/69 what happens when 3d network?

12/2/69 Overall BVP for Neyludes
M order boundary
Statement of whole problem
& some intuitive statements

12/3/69 Steady State Diagram
& Hand Calc. St. St. Graph
 V_h & V_{hs} vs R_h/R_{hin} 10^{-2} to 10^{+2}
for $R_h/R_{hin} = 10$

Worked out image & symmetry approach
explicitly, with diagrams, for
AAS - Boston presentation.

12/7/69 I wrote out steady state solutions from ^{made an error} superposition

12/22/69 Ruizell wrote out solutions in Laplace transform form
for gliders with $\left\{ \begin{array}{l} \text{sealed end} \\ \text{clamped end} \end{array} \right.$

2/3/70 Recap steady state solutions

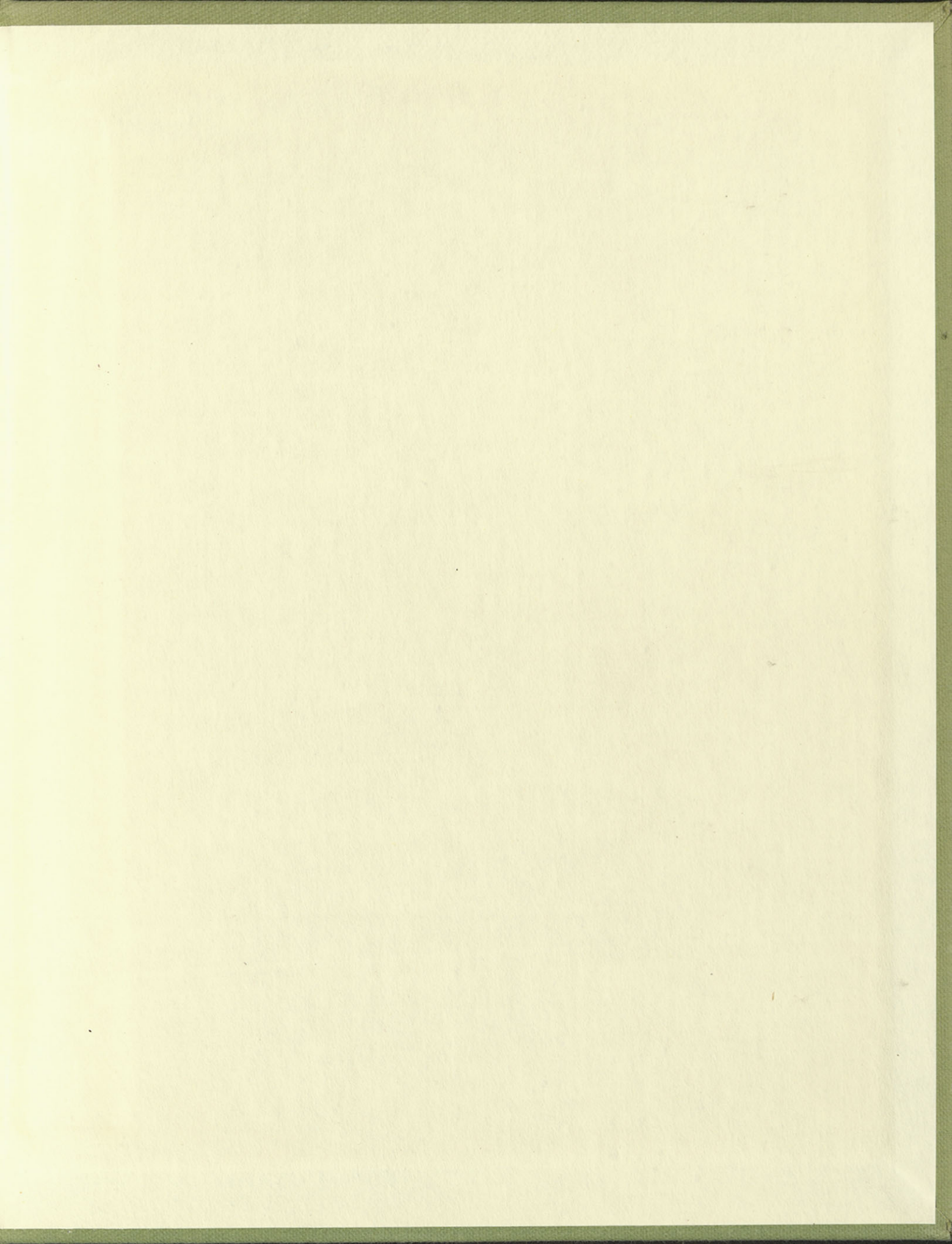
2/4/70 Begin work on outlines of two papers

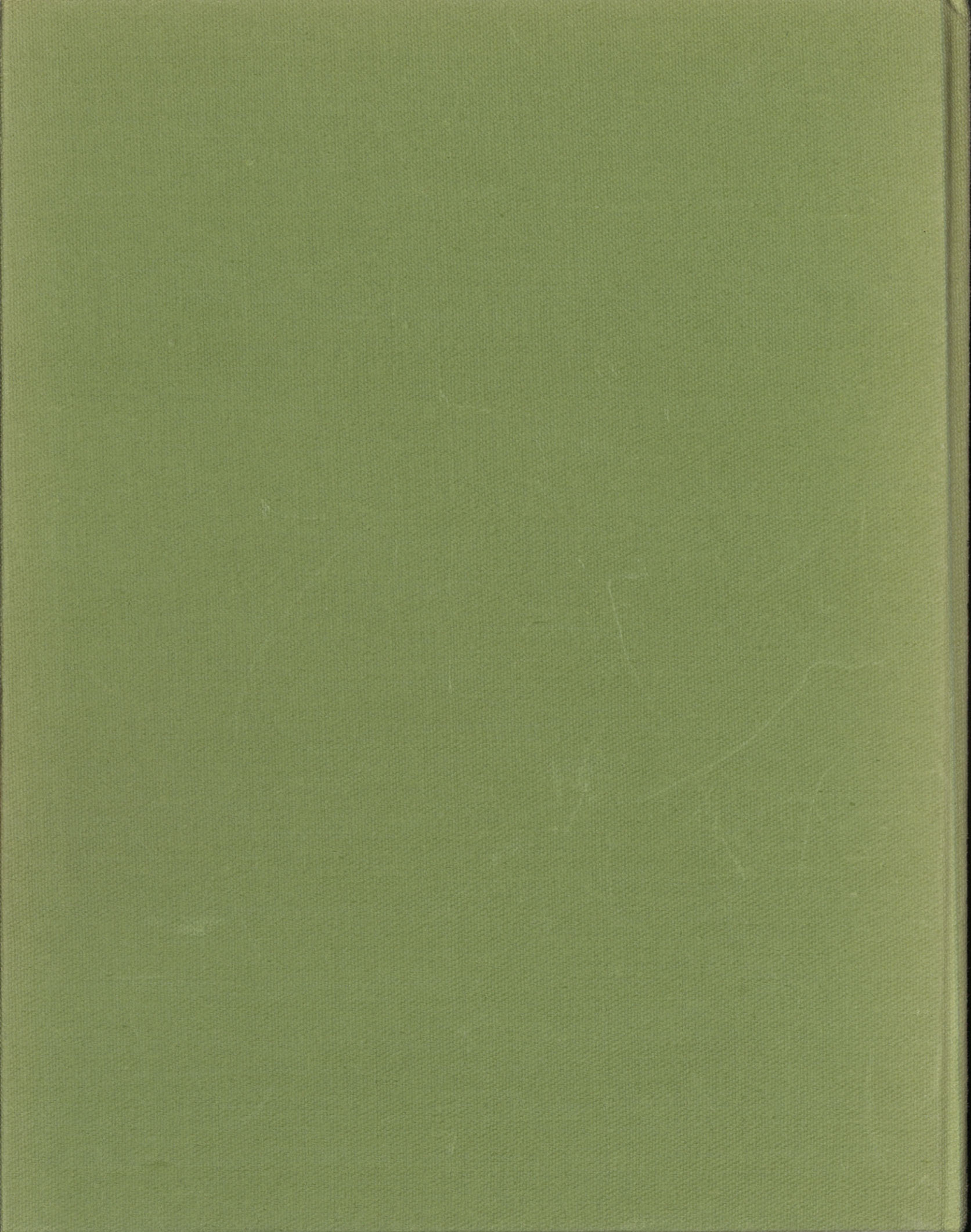
Computed results for current injection

John got results & worked on machine plots

2/10/70 Working outline for papers

2/11/70 Started draft of Section II of Paper A
Steady State Solution for current injection

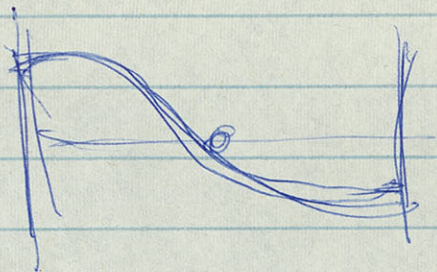




$$C_0 = \frac{1}{L} \int_0^L F(x) dx$$

$$C_1 = \frac{2}{L} \int_0^L F(x) \cos(\pi x/L) dx$$

$$\therefore \frac{\left[2 \left(\frac{L}{\pi} \right) \sin \left(\frac{\pi x}{L} \right) \right]_A^L}{L-A} = \frac{-2 \frac{L}{\pi} \sin(\pi A/L)}{L-A}$$



Peaks for Series

let $n = m$

$$\frac{1}{2} \int e^{-Kx} \frac{(-K \cos 2mx + 2m \sin 2mx)}{K^2 + 4m^2(\frac{\pi}{L})^2} dx$$

$$\rightarrow \frac{e^{-Kx}(-K + 0)}{K^2 + 0}$$

0.5L

$$\frac{1}{2} \left(\frac{e^{-KL}(-K \cos 2mL + 0) + K}{K^2 + 4m^2(\frac{\pi}{L})^2} + \frac{-Ke^{-KL} + K}{K^2} \right)$$

$$= \frac{K}{2} \left[\frac{-e^{-KL} + 1}{K^2 + 4m^2(\frac{\pi}{L})^2} + \frac{-e^{-KL} + 1}{K^2} \right]$$

$$= \frac{K}{2} (1 - e^{-KL}) \left(\frac{2K^2 + 4m^2(\frac{\pi}{L})^2}{K^2(K^2 + 4m^2(\frac{\pi}{L})^2)} \right)$$

$$K^2 + 2m^2 \left(\frac{\pi}{L} \right)^2$$

$$K^2 + 4m^2 \left(\frac{\pi}{2L} \right)^2$$

$$\int_0^L e^{-Kx} \cos nx \cos mx dx$$

Becomes $\cos mx \cos nx = \frac{1}{2} \cos(n+m)x + \frac{1}{2} \cos(n-m)x$

$$\frac{1}{2} \int_0^L e^{-Kx} \cos(n+m)x + e^{-Kx} \cos(n-m)x$$

$$\frac{1}{2} \left[\frac{e^{-Kx} (-K \cos(n+m)x + (n+m) \sin(n+m)x)}{K^2 + (n+m)^2} + \frac{e^{-Kx} (-K \cos(n-m)x + (n-m) \sin(n-m)x)}{K^2 + (n-m)^2} \right]$$

$$\frac{e^{-KL} (n-m) \sin(n-m)L}{K^2 + (n-m)^2}$$

change m to $n\pi x/L$
 n to $n\pi x/L$

$$\frac{1}{2} \left(\frac{e^{-KL} (-K \cos(n+m)\pi + (n+m)\pi \sin(n+m)\pi)}{K^2 + (n+m)^2 (\frac{\pi}{L})^2} + K \right)$$

$$+ \frac{e^{-KL} (-K \cos(n-m)\pi + 0)}{K^2 + (n-m)^2 (\frac{\pi}{L})^2} + K$$

$$\frac{K}{2} \left[\frac{-(-1)^{n+m} e^{-KL} + 1}{K^2 + (n+m)^2 (\frac{\pi}{L})^2} + \frac{-(-1)^{n-m} e^{-KL} + 1}{K^2 + (n-m)^2 (\frac{\pi}{L})^2} \right]$$

Because $\int e^{ax} \cos bx \, dx = \frac{e^{ax} (a \cos bx + b \sin bx)}{a^2 + b^2}$

It follows that for sin, ~~at~~ $\sin - \cos$

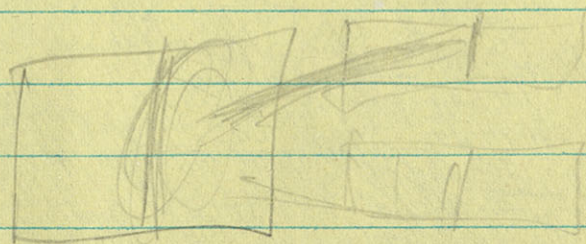
$$\int_0^L e^{Kx/2} \left\{ \cos \frac{m\pi x}{L} + \frac{KL}{2m\pi} \sin \frac{m\pi x}{L} \right\} dx \Rightarrow$$

$$\left[\frac{e^{Kx/2} \left(\frac{K}{2} \cos \frac{m\pi x}{L} + \frac{m\pi}{L} \sin \frac{m\pi x}{L} \right)}{\left(\frac{K}{2} \right)^2 + \left(\frac{m\pi}{L} \right)^2} + \frac{e^{Kx/2} \left(\frac{K^2 L}{4m\pi} \sin \frac{m\pi x}{L} - \frac{KL}{2m\pi} \left(\frac{K}{2} \right) \cos \frac{m\pi x}{L} \right)}{\left(\frac{K}{2} \right)^2 + \left(\frac{m\pi}{L} \right)^2} \right]_0^L$$

$$\frac{q \alpha r^2}{c \alpha r}$$

$$\frac{q \alpha r}{c}$$

C_i
 G_{ij}



Should sit down with Jim sometime to get correct expression for G_{ij} damping at branch points